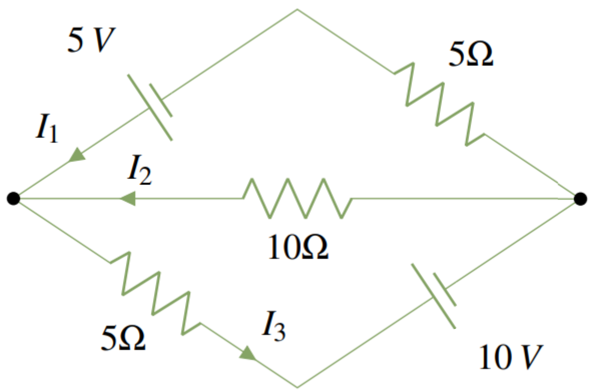
Name: Solutions
Start Time:
End Time:
Date:

Math 260
Quiz 2 (25 min)

1. (5 points) Find all the currents in the circuit below. Solve by row reducing an augmented matrix. You may use your calculator to do the row reducing in this problem.



Left Junction: $I_1 + I_2 = I_3$

Right Junction: $I_1 + I_2 = I_3$ ⇐ redundant

Top loop: $5 + 10 I_2 - 5 I_1 = 0$

Bottom loop: $10 + 5 I_3 + 10 I_2 = 0$

I_1	I_2	I_3	constants
I_1 +	I_2 −	I_3	$= 0$
$5I_1$ −	$10I_2$		$= 5$
	$10 I_2$ +	$5I_3$	$= -10$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 5 & -10 & 0 & 5 \\ 0 & 10 & 5 & -10 \end{array}\right] \xrightarrow{RREF} \begin{array}{c} \begin{array}{ccc} I_1 & I_2 & I_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & -0.2 \\ 0 & 1 & 0 & -0.6 \\ 0 & 0 & 1 & -0.8 \end{array}\right] \end{array}$$

$I_1 = -0.2$ Amps
$I_2 = -0.6$ Amps
$I_3 = -0.8$ Amps

2. (4 points) Prove: $\forall\ x \in \mathbb{R}$, if $x \notin [2,3]$ then $(x-2)(x-3) > 0$

proof: Let $x \in \mathbb{R}$ and suppose $x \notin [2,3]$.

case 1: Suppose $x < 2$.

Subtract 2 $\Rightarrow$ $x-2 < 0$, so $x-2$ is negative.

Subtract 1 $\Rightarrow$ $x-3 < -1$ and since $-1 < 0$ $\Rightarrow$ $x-3 < 0$, so $x-3$ is negative

Then $(x-2)(x-3) = (\text{neg.})(\text{neg.}) = \text{pos.}$, so $(x-2)(x-3) > 0$.

case 2: Suppose $x > 3$.

Subtract 2 $\Rightarrow$ $x-2 > 1$ and $1 > 0$, so $x-2 > 0$, so $x-2$ positive

Subtract 1 $\Rightarrow$ $x-3 > 0$ so $x-3$ is positive.

Then $(x-2)(x-3) = (\text{pos.})(\text{pos.}) = \text{pos.}$, so $(x-2)(x-3) > 0$.

3. (1 point) Disprove: $\forall\ x \in \mathbb{R}$, if $x > 0$ then $\sin x > x$

Let $x = \pi$. Then $\pi > 0$, but $\sin \pi \not> \pi$

bec. $0 \not> \pi$.

Extra Credit (5 points): Solve the following system of equations. Row reduce by hand, do not use a calculator, and only do one row operation at a time.

$$\begin{aligned} x_1 + 2x_2 - x_3 + 2x_4 + x_5 &= 0 \\ x_1 + 2x_2 + 2x_3 \phantom{{}+ 2x_4} + x_5 &= 0 \\ 2x_1 + 4x_2 - 2x_3 + 3x_4 + x_5 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 1 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 2 & 4 & -2 & 3 & 1 & 0 \end{array}\right] \xrightarrow{-1R_1+R_2 \to R_2} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 1 & 0 \\ 0 & 0 & 3 & -2 & 0 & 0 \\ 2 & 4 & -2 & 3 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{-2R_1+R_3 \to R_3} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 1 & 0 \\ 0 & 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 \end{array}\right] \xrightarrow{\frac{1}{3}R_2 \to R_2} \left[\begin{array}{ccccc|c} 1 & 2 & -1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -2/3 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{1R_2+R_1 \to R_1} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 4/3 & 1 & 0 \\ 0 & 0 & 1 & -2/3 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 \end{array}\right] \xrightarrow{-1R_3 \to R_3} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 4/3 & 1 & 0 \\ 0 & 0 & 1 & -2/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{-4/3R_3+R_1 \to R_1} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & -2/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{\frac{2}{3}R_3+R_2 \to R_2} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \end{array}$$

$x_2 = r \qquad x_5 = t$

top eqn $\Rightarrow$ $x_1 + 2x_2 - \frac{1}{3}x_5 = 0 \Rightarrow x_1 = \frac{1}{3}t - 2r$

middle eqn $\Rightarrow$ $x_3 + \frac{2}{3}x_5 = 0 \Rightarrow x_3 = -\frac{2}{3}t$

bottom eqn $\Rightarrow$ $x_4 + x_5 = 0 \Rightarrow x_4 = -t$

$$\boxed{\text{Solution Set} = \left\{ \left(\tfrac{1}{3}t - 2r,\ r,\ -\tfrac{2}{3}t,\ -t,\ t \right) \;\middle|\; r, t \in \mathbb{R} \right\}}$$